Momentum

- Momentum is what Newton called the "***quantity of motion***" of an object.
- Also called "***Mass in motion***"

$$p = mv$$

p = momentum
m = mass
v = velocity

The units for momentum are: $\frac{kg \cdot m}{s}$

Momentum

- Momentum is affected by mass and velocity
- A speeding bullet may have the same momentum as a slow moving car
- The more momentum an object has, the more force will be needed to change its motion

Example: It is hard to stop a large, fast train

What is the momentum of a 2200 kg car driving at 26 m/s?

m = 2200 kg v = 26 m/s

$$p = mv$$

$$p = (2200\,kg)(26\,m/s) \quad \boxed{= 57000 \frac{kg \cdot m}{s}}$$

3

Impulse

- Impulse is the change in momentum
- Impulse relates the time and force needed to change an objects momentum

More contact time → Less force needed

Less contact time → Less force needed

4

Impulse – Momentum Theorem

I = impulse, but sometimes J is used.

$$\text{Impulse} = F\Delta t = p_f - p_i$$

The units for impulse are:

$N \cdot s$ or $\frac{kg \cdot m}{s}$

Deriving Impulse

- We know:

$$F = ma \qquad a = \frac{\Delta v}{t}$$

- Then substitute and rearrange

$$F = ma = m\frac{\Delta v}{t} \longrightarrow \boxed{F \cdot t = \underbrace{m\Delta v}_{\Delta p}}$$

Impulse Examples

- What should a boxer do when he gets hit?

$$F \cdot t = \Delta p$$

When a boxer "rolls with the punch" he increases the contact time of the punch.

This makes the force much smaller.

7

More Impulse Examples

- Dropping an egg on a pillow will increase the time of the momentum change. This lowers the force.
- A car uses an airbag to increase the time the head changes its momentum during an accident.
- A car driving into a wall will change its momentum very fast, resulting in a huge force.
- A car hitting the brakes and slowing down gradually will result in very low forces.

8

Impulse – Momentum Theorem

$$I = F \cdot t = \Delta p = m(v_f - v_i)$$

I = impulse
F = force
t = time

Δp = change momentum
m = mass
v_f = final velocity
v_i = initial velocity

Common units for impulse are: $N \cdot s$

What force does it take to stop a 2200 kg car driving at 26 m/s?

Case 1 – Use brakes for 21 seconds
Case 2 – Crash into a wall – Stop in 0.22 s

$$F\Delta t = \Delta p = m(v_f - v_i)$$

$$\Delta p = 2200\,kg\,(0\,m/s - 26\,m/s)$$

$$\Delta p = -57000\frac{kg \cdot m}{s}$$

10

What force does it take to stop a 2200 kg car driving at 26 m/s?

$$F\Delta t = \Delta p \longrightarrow F = \frac{\Delta p}{\Delta t}$$

Case 1 – Use brakes for 21 seconds

$$F = \frac{-57000\,kg \cdot m/s}{21\,s} = -2700\,N$$

11

What force does it take to stop a 2200 kg car driving at 26 m/s?

Case 2 – Crash into a wall – Stop in 0.22 s

$$F = \frac{-57000\,kg \cdot m/s}{0.22\,s} = -260000\,N$$

It is typically safer to use brakes to stop a car instead of using walls.

12

Conservation of Momentum

- Total momentum in a system does not change.
- Momentum is transferred between objects
  - Example Equation (depends on situation)

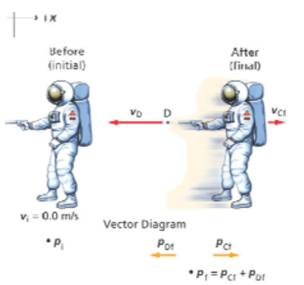
$$p_{Ai} + p_{Bi} = p_{Af} + p_{Bf}$$

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

Conservation of Momentum

- Each problem must be analyzed to figure out what is going on. Do objects:
  - bounce away in opposite directions?
  - Stick together? (Car crash)
  - Separate? (Bullet giving recoil)
  - Transfer momentum? (Pool ball)
- Choose an appropriate coordinate system to describe the velocity of each object (+/-)

An astronaut at rest in space fires a thruster pistol that expels 35 g of hot gas at 875 m/s. The combined mass of the astronaut and pistol is 84 kg. How fast and in what direction is the astronaut moving after firing the pistol?



$$m_A = 0.035\,kg$$
$$m_B = 84\,kg$$
$$v_{Ai} = 0\,m/s$$
$$v_{Bi} = 0\,m/s$$
$$v_{Af} = 875\,m/s$$

- Objects separate in opposite directions
- Develop equations to use

$$p_{ABi} = p_{Af} + p_{Bf}$$

- The initial terms drop out since v_{ABi} = 0 m/s

$$0 = m_A v_{Af} + m_B v_{Bf}$$

$$m_A v_{Af} = -m_B v_{Bf}$$

- Solve for v_{Bf}

$$v_{Bf} = -\frac{m_A v_{Af}}{m_B}$$

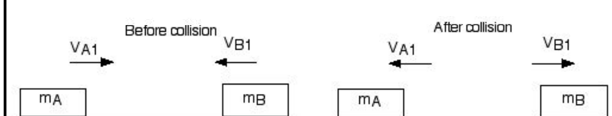
- Plug and Chug

$$v_{Bf} = -\frac{0.035\,kg \cdot 875\,m/s}{84\,kg} = -0.36\,m/s$$

- Negative velocity means opposite direction

Elastic Collision

- **This occurs when cars bounce off each other.**
- The distance the cars bounce depends on the original momentum

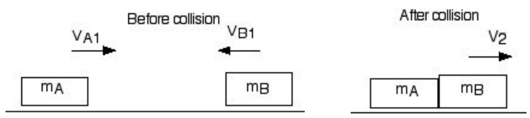


$$p_{Ai} + p_{Bi} = p_{Af} + p_{Bf}$$

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

Inelastic Collision

- **This occurs when cars stick together.**
- The cars will combine and move in the direction of the car with greater momentum

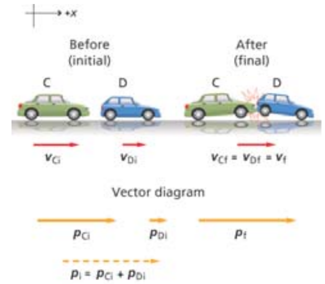


$$p_{Ai} + p_{Bi} = p_{ABf}$$

$$m_A v_{Ai} + m_B v_{Bi} = (m_A + m_B) v_{ABf}$$

19

A 1875-kg car going 23 m/s rear-ends a 1025-kg compact car going 17 m/s on ice in the same direction. The two cars stick together. How fast do the two cars move together immediately after the collision?



$$m_A = 1875\,kg$$

$$m_B = 1025\,kg$$

$$v_{Ai} = 23\,m/s$$

$$v_{Bi} = 17\,m/s$$

20

- Objects combine, continue in same direction
- Develop equations to use

$$p_{Ai} + p_{Bi} = p_{ABf}$$

$$m_A v_{Ai} + m_B v_{Bi} = (m_A + m_B) v_{ABf}$$

- Solve for v_{ABf}

$$v_{ABf} = \frac{m_A v_{Ai} + m_B v_{Bi}}{m_A + m_B}$$

21

- Plug and Chug

$$m_A v_{Ai} = 1875\,kg \cdot 23\,m/s = 43125\,kg \cdot m/s$$

$$m_B v_{Bi} = 1025\,kg \cdot 17\,m/s = 17425\,kg \cdot m/s$$

$$v_{ABf} = \frac{43125\,kg \cdot m/s + 17425\,kg \cdot m/s}{1875\,kg + 1025\,kg}$$

$$\boxed{v_{ABf} = 20.8\,m/s}$$

22

Momentum in Two Dimensions

- Each momentum (or velocity) must be broken to x and y components (sin, cos, etc)
- Sum component momenta (velocities) and find the resultant momentum (velocity)
- Remember to use p = mv for each momentum. Use whichever form, p or mv, that is more useful.

23

Momentum in Two Dimensions

Example – 2 objects, A and B, crashing into each other from different directions

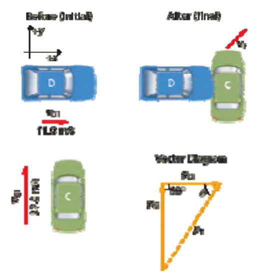
$p_{Ai,x} + p_{Bi,x} = P_{ABf,x}$ $p_{Ai,y} + p_{Bi,y} = P_{ABf,y}$

Resultant Momentum $$p_{ABf} = \sqrt{p_{ABf,x}^2 + p_{ABf,y}^2}$$

Angle from x-axis $$\theta = \tan^{-1}\left(\frac{P_{ABf,y}}{P_{ABf,x}}\right)$$

24

A 1325-kg car, A, moving north at 27.0 m/s, collides with a 2165-kg car, B, moving east at 11.0 m/s. The two cars are stuck together. In what direction and with what speed do they move after the collision?



$$m_A = 1325\,kg$$
$$m_B = 2165\,kg$$
$$v_{Ai} = 27\,m/s$$
$$v_{Bi} = 11\,m/s$$

- Find total momentum for each component

$$p_{Ai,x} + p_{Bi,x} = p_{ABf,x}$$
$$(1325\,kg)(27\,m/s) + 0 = 35775\,kg \cdot m/s$$

$$p_{Ai,y} + p_{Bi,y} = p_{ABf,y}$$
$$0 + (2165\,kg)(11\,m/s) = 23815\,kg \cdot m/s$$

- Find resultant momentum

$$p_{ABf} = \sqrt{{p_{ABf,x}}^2 + {p_{ABf,y}}^2}$$
$$p_{ABf} = \sqrt{(23815\,kg \cdot m/s)^2 + (35775\,kg \cdot m/s)^2}$$
$$p_{ABf} = 42976.8\,kg \cdot m/s$$

- Relate momentum to velocity

$$p_{ABf} = (m_A + m_B)v_{ABf}$$

- Solve for v_{ABf}

$$v_{ABf} = \frac{p_{ABf}}{m_A + m_B} = \frac{42976.8\,kg \cdot m/s}{1325\,kg + 2165\,kg}$$

$$\boxed{v_{ABf} = 12.3\,m/s}$$

- Finding Direction

$$\theta = \tan^{-1}\left(\frac{35775\,kg \cdot m/s}{23815\,kg \cdot m/s}\right) = 56.4°\ N\ of\ E$$